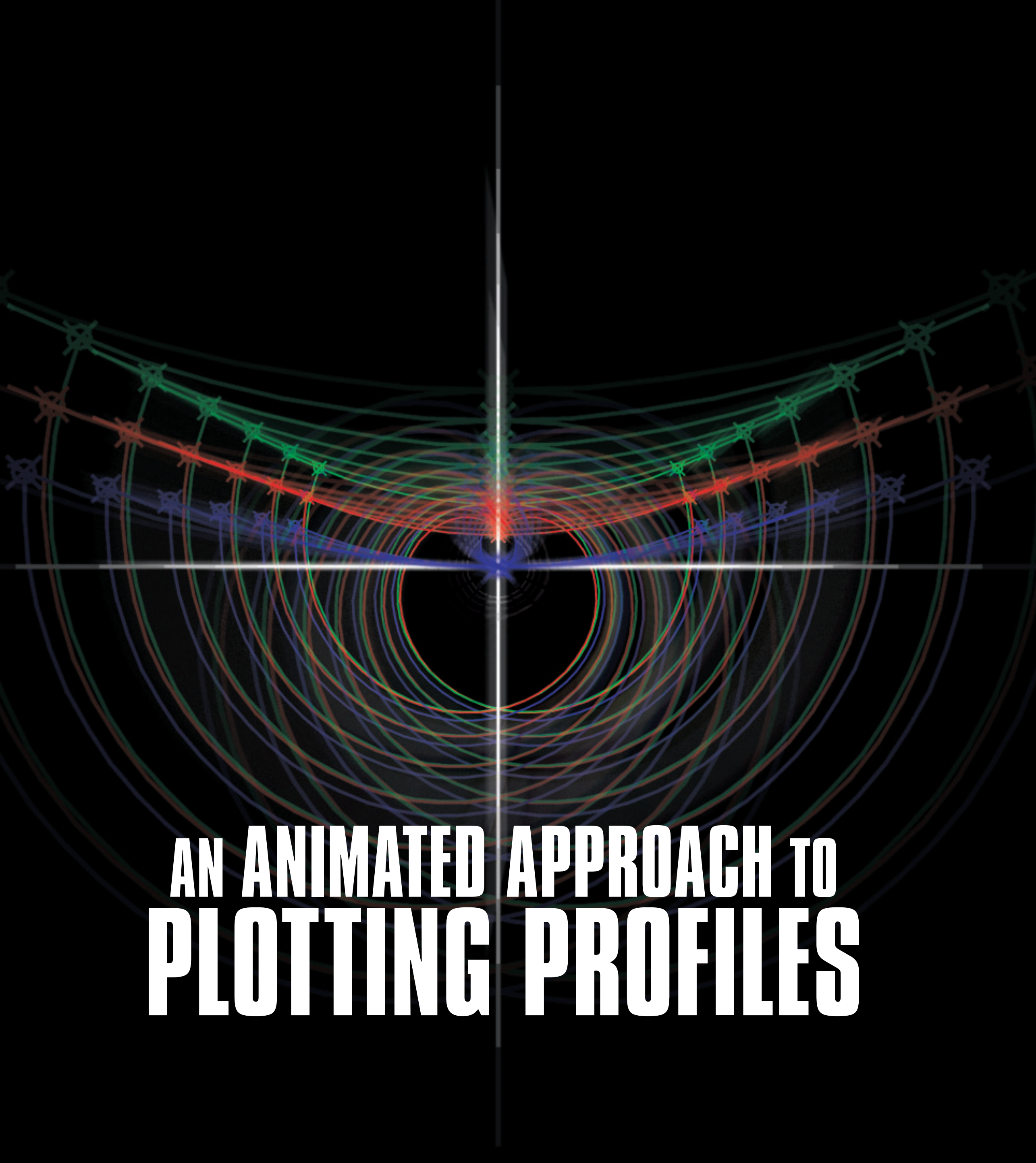

AN ANIMATED APPROACH TO PLOTTING PROFILES

ANIMATED SIMULATIONS OF TROCHOID SPIRAL AND EPITROCHOIDAL SPIRAL CURVES ARE AN ELEGANT AND USEFUL MEANS OF DISPLAYING THE INHERENT GEOMETRICAL PROPERTIES FOR GEAR DESIGN.

By Sandor J. Baranyi

Trochoid Spiral (TS) and Epitrochoidal Spiral (ETS) curves and special flash animations were developed by the author in 2004 and 2005 for nonintersecting and intersecting right angle axis gear drives involving R/L (rotary-to-linear) and L/R motion conversion, and for parallel shaft R/R motion conversion, respectively.

The design of TS and ETS profile gears is based on cylindrical tooth followers in a toothed engagement with members of spiral groove type face gears. TS gearing represents a simple, limiting case of ETS gearing. TS and ETS curves represent pitch curves of corresponding gear profiles.

We distinguish three types each of TS and ETS curves as L-TS (looped), Z-TS (zero offset) and P-TS (prolate) types as in Figure 1A, and L-ETS, Z-ETS, and P-ETS, as illustrated in Figure 2A.

As in the case of cutting traditional hypotrochoid, epitrochoid, and trochoid gears, CNC gear cutting technology (milling, wire EDM, etc.) represents an excellent choice for high-precision TS and ETS parts for operation and service features that can seldom be matched by other gear cutting types. This also translates into low parts cost, short lead time for prototype and pilot production quantities, and for tooling.

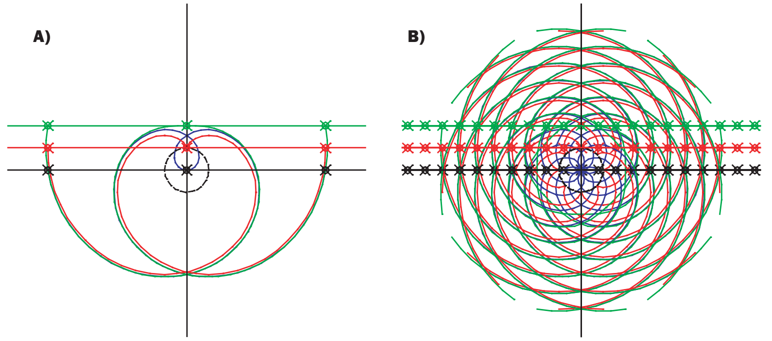

FIGURE 1: Sample TS curves: L-TS (blue), Z-TS (red), P-TS (green)

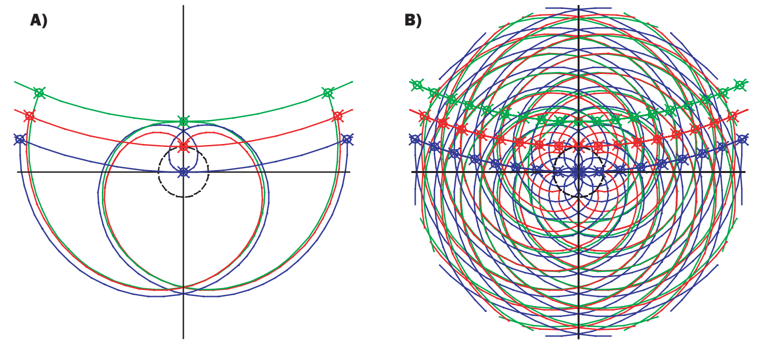

FIGURE 2: Sample ETS curves: L-ETS (blue), Z-ETS (red), P-ETS (green)

Parametric equations for TS curves are as follows:

Equation 1: $x = e\ q\ \cos(q) - w\ \sin(q)$
Equation 2: $y = e\ q\ \sin(q) + w\ \cos(q)$

where $\pi=3.1415$, $-2\pi\ e \leq q \leq 2\pi\ e$ and q is the independent variable in radians, e is the radius of pitch circle for the sample curves shown in dashed line with center at x=y=0 in Figure 1A. The sample curves depend on the particular value of design constant w=y for x=q=0.

- w=0 for the L-TS curve, also known as the Archimedes Spiral (AS). In the author's opinion the AS is by far the first mathematically exact gear profile that was discovered and formulated by the Greek mathematician and physicist Archimedes 2,300 years ago using geometry, prior to the existence of algebra and the concept of numeral zero.

There have been many important inventions using the AS over the centuries, notably the scroll-chuck for lathes and related machines, and using the AS in special mechanisms such as mechanical analog squaring devices for computing the squares of numbers to high digital accuracies, just to mention two important examples.

In reference to the opening paragraph, the L-TS is the only intersecting axis rectangle drive within the scope of this article.

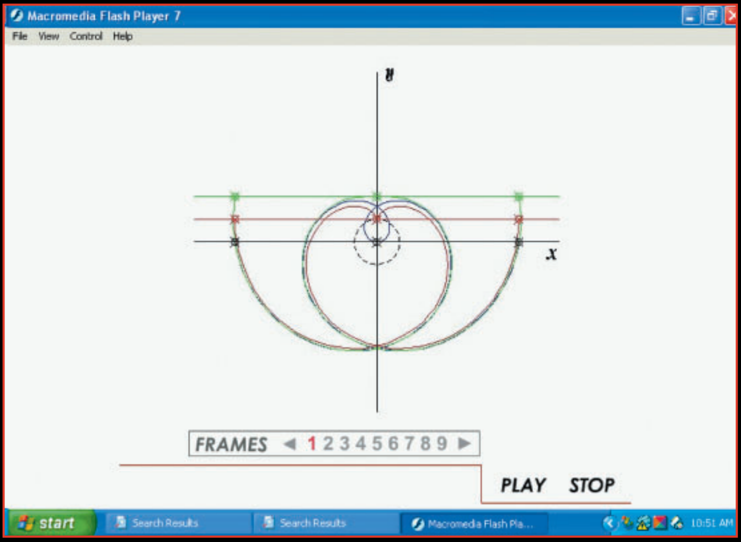

FIGURE 3: ACADSvF flash animation for TS curves

- w=e for the Z-TS curve, implying that the pole (cusp) of the curve for x=q=0 coincides with the topmost point of the pitch circle in Figure 1A,

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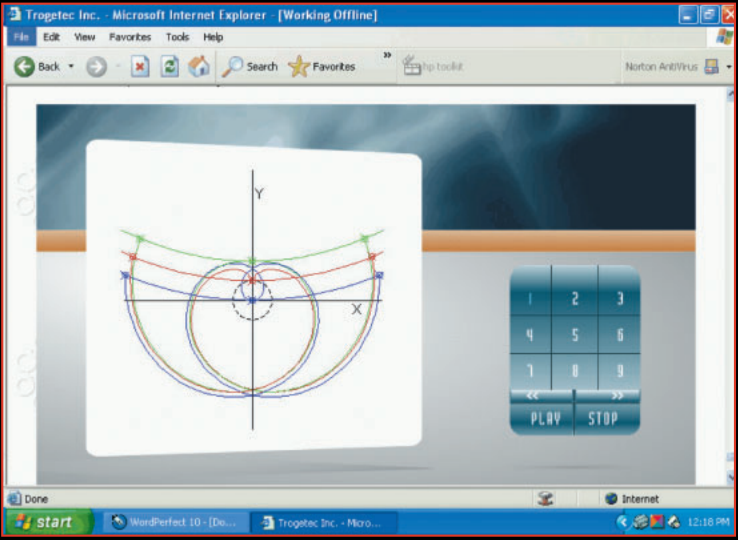


FIGURE 4: ACADSvF flash animation for ETS curves

so that w-e=0, hence the designation of zero offset. The curve is also known as the involute of a circle, a special case of TS profiles.

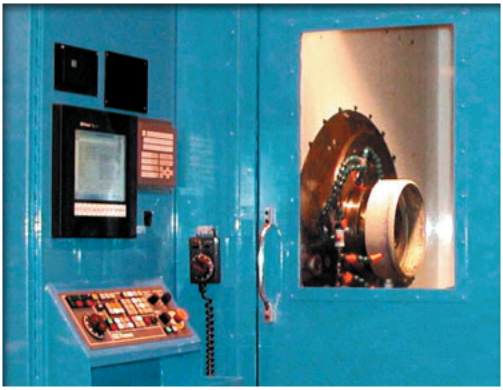
The formulation of traditional involute gearing by the Swiss mathematician Leonhard Euler has had a profound influence on watchmaking and other industries for centuries.

- w=2e for the sample P-TS curve. The new P-TS profiles offer important gear design advantages over the two other profile types because they are least affected by certain profile undercut and insufficient radius of curvature of gear profile surface conditions, and therefore they do not require extra design measures that could prove counterproductive. Through-shaft gear design, versus cantilever shaft support for the gear, may also be realized as an advantage for certain applications.

Parametric equations for ETS curves are as follows:

Equation 3: $x = -(b \sin(q) - Q \sin(b\,q))\, n\, e$
Equation 4: $y = (b \cos(q) - Q \cos(b\,q))\, n\, e$

where $-2\pi\, e \le q \le 2\pi\, e$ and q is the independent variable in radians, e is the radius of pitch circle for the three sample curves as shown as dashed lines in Figure 2A with center at x=y=0, while n designates the number of rollers in the follower, and b and Q are constants depending on the particular value of w=y for x=q=0, just like in the examples above for TS curves. For n=16 for the ETS curves in Figure 2A, the following information applies:



- $w=0$, $b=Q=1.0625$ for the L-ETS curve. Substituting b and q into equations 3 and 4, data points for the curve were obtained for plotting it as shown in Figure 2A, where the follower center coordinates of $x=0$ and $y=(n+1)e=17e$ apply for all three concentric pitch circles. Consequently, the pitch radius of the roller follower for the L-ETS curve is 17e.

- $w=e$, $b=1.0625$, and $Q=1$ for the Z-ETS curve as shown in Figure 2A. We note that the pitch radius of the follower in red is 16e in this case.

- $w=2e$, $b=1.0625$, and $Q=.9375$ for the P-ETS curve and the corresponding follower of 15e pitch radius as shown in Figure 2A.

Calculation of the gear ratio for TS and ETS gears is simple: one full turn of the gear corresponds to one linear or circular pitch, P, respectively, as $P=2\pi e$, where e designates the pitch circle radius for the gear.

As an example, a given ETS gear train uses a follower of $n=10$ rollers for a gear ratio, R, as, $R=-n:1=-10:1$, where the negative sign indicates that the gear and follower rotate in opposite directions.

> **“THIS ANIMATION TECHNIQUE WAS PRIMARILY DEVELOPED FOR THE NEW TS, ETS, AND OTHER GEAR CATEGORIES WHERE THE USE OF VISUALS, PROTOTYPES, AND DEMO UNITS CAN BE OF PARAMOUNT IMPORTANCE FOR THE BEST RESULTS.”**

ACADSvF (animated CAD Solutions version F by Trogetec, using flash by Macromedia) animation for Figures 1 and 2 (using eight increments of 45-degree gear rotation as shown) is provided on the Web site listed below. The corresponding composite plots in Figures 1B and 2B serve as transition-type visuals between Figures 1A and 2A on the one hand, and the corresponding flash animations on the other. For monitor screenshots of the shockwave flash animations we refer to Figures 3 and 4.

This animation technique was primarily developed for the new TS, ETS, and other gear categories, for design studies by individuals or small groups of design/mfg/QC engineers, and for executive project reviews, when the use of visuals, prototypes, and demo units can be of paramount importance for the best results.

ABOUT THE AUTHOR:

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