Determining Gear Efficiency

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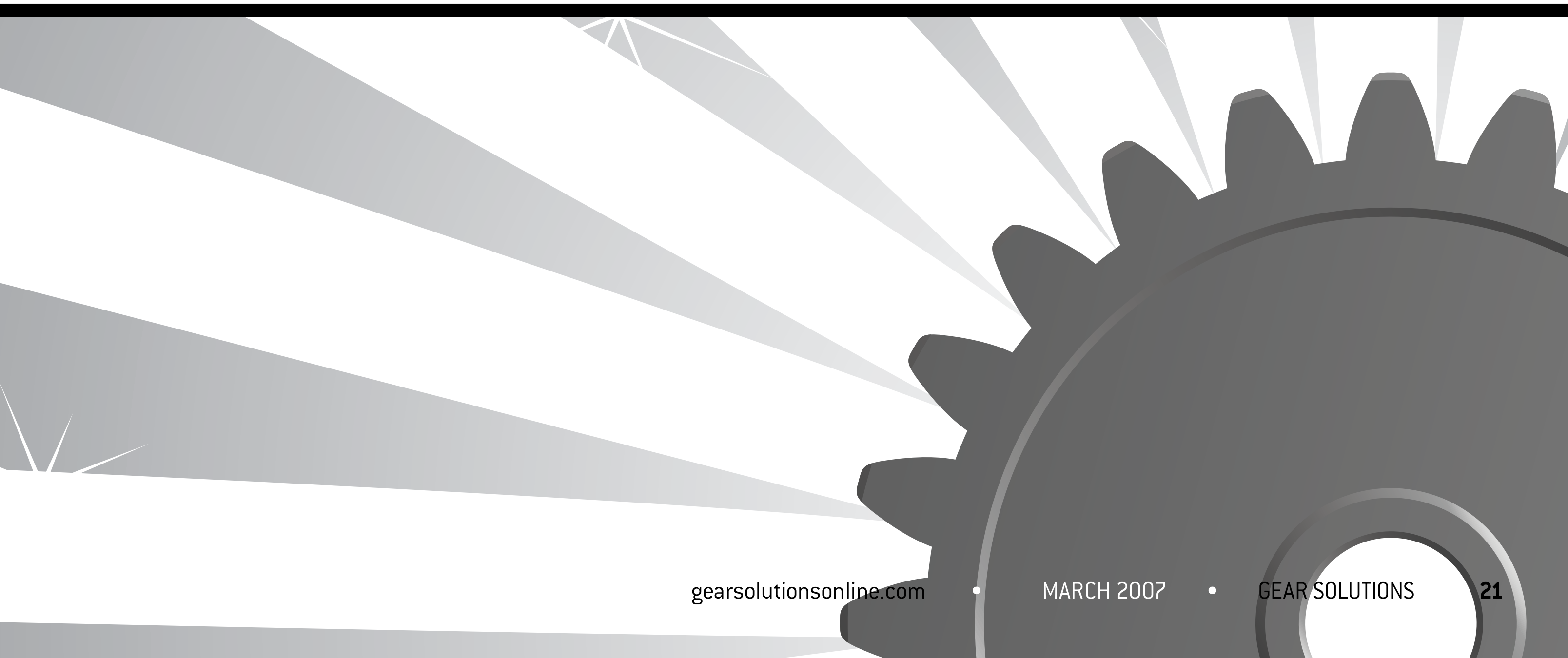
By analyzing certain parameters, the authors present an original method for determining the efficiency of gears.

Abstract: This paper presents an original method for determining the efficiency of the gear. The originality of this method relies on the eliminated friction modulus. In the following pages we analyze the influence of a few parameters concerning gear efficiency. These parameters are: z_1—the number of teeth for the primary wheel of gear; z_2—the number of teeth of the secondary wheel of gear; α_o—the normal pressure angle on the divided circle; and β—the inclination angle. With the relations presented in this paper, one can synthesize the gear's mechanisms. Today, gears are present everywhere in the mechanical world, especially the automotive, electronics, and energy industries, etc. By optimizing this mechanism, one can improve the functionality of transmission gears.



Introduction

In presenting this original method for calculating the efficiency of the gear, the originality consists in the way of determining the gear's efficiency, because we haven't used the friction forces involved in coupling—this new way eliminates the classical method, in fact. The necessity of determining the friction coefficients by different experimental methods is eliminated, as well. The efficiency determinates by the new method are the same as in the classical method; namely, the mechanical efficiency of the gear. Precisely, one determines the dynamics efficiency, but at the transmission's gears, the dynamics efficiency is the same as the mechanical efficiency.

Determining the Momentary Dynamic (Mechanical) Efficiency

The calculating relations [2, 3], are the next (1-21), (see fig. 1):

$$\begin{cases} F_\tau = F_m \cdot \cos\alpha_1 \\ F_\psi = F_m \cdot \sin\alpha_1 \\ v_2 = v_1 \cdot \cos\alpha_1 \\ v_{12} = v_1 \cdot \sin\alpha_1 \\ \overline{F}_m = \overline{F}_\tau + \overline{F}_\psi \\ \overline{v}_1 = \overline{v}_2 + \overline{v}_{12} \end{cases} \qquad (1)$$

with: F_m - the motive force (the driving force);
F_τ - the transmitted force (the useful force);
F_ψ - the slide force (the lost force);
v_1- the velocity of element 1, or the speed of wheel 1 (the driving wheel);
v_2- the velocity of element 2, or the speed of wheel 2 (the driven wheel);
v_{12}- the relative speed of the wheel 1 in relation with the wheel 2 (this is a sliding speed).

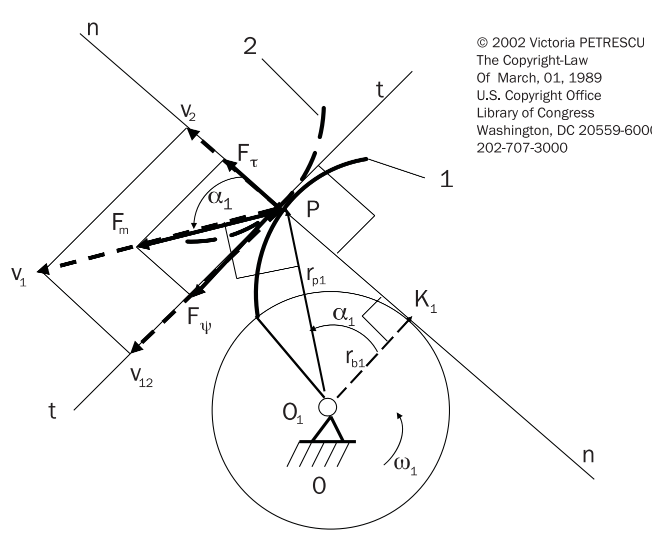


FIGURE 1: THE FORCES OF THE GEAR

The consumed power (in this case the driving power):

$$P_c \equiv P_m = F_m \cdot v_1 \qquad (2)$$

The useful power (the transmitted power from the profile 1 to the profile 2) will be written:

$$P_u \equiv P_\tau = F_\tau \cdot v_2 = F_m \cdot v_1 \cdot \cos^2 \alpha_1 \qquad (3)$$

The lost power will be written:

$$P_\psi = F_\psi \cdot v_{12} = F_m \cdot v_1 \cdot \sin^2 \alpha_1 \qquad (4)$$

The momentary efficiency of couple will be calculated directly with the next relation:

$$\begin{cases} \eta_i = \dfrac{P_u}{P_c} \equiv \dfrac{P_\tau}{P_m} = \dfrac{F_m \cdot v_1 \cdot \cos^2 \alpha_1}{F_m \cdot v_1} \\ \eta_i = \cos^2 \alpha_1 \end{cases} \qquad (5)$$

The momentary losing coefficient [1], will be written:

$$\begin{cases} \psi_i = \dfrac{P_\psi}{P_m} = \dfrac{F_m \cdot v_1 \cdot \sin^2 \alpha_1}{F_m \cdot v_1} = \sin^2 \alpha_1 \\ \eta_i + \psi_i = \cos^2 \alpha_1 + \sin^2 \alpha_1 = 1 \end{cases} \qquad (6)$$

One can easily see that the sum of the momentary efficiency and the momentary losing coefficient Now one can determine the geometrical elements of the gear. These elements will be used in determining the couple efficiency, η.

The Geometrical Elements of the Gear

One determines the next geometrical elements of the external gear, [2,3], (for the right teeth, β=0):

The radius of the basic circle of wheel 1 (of the driving wheel), (7):

$$r_{b1} = \frac{1}{2} \cdot m \cdot z_1 \cdot \cos\alpha_0 \qquad (7)$$

The radius of the outside circle of wheel 1 (8):

$$r_{a1} = \frac{1}{2} \cdot (m \cdot z_1 + 2 \cdot m) = \frac{m}{2} \cdot (z_1 + 2) \qquad (8)$$

One determines now the maximum pressure angle of the gear (9):

$$\cos\alpha_{1M} = \frac{r_{b1}}{r_{a1}} = \frac{\frac{1}{2} \cdot m \cdot z_1 \cdot \cos\alpha_0}{\frac{1}{2} \cdot m \cdot (z_1 + 2)} = \frac{z_1 \cdot \cos\alpha_0}{z_1 + 2} \qquad (9)$$

And now one determines the same parameters for the wheel 2, the radius of basic circle (10) and the radius of the outside circle (11) for the wheel 2:

$$r_{b2} = \frac{1}{2} \cdot m \cdot z_2 \cdot \cos\alpha_0 \qquad (10)$$

$$r_{a2} = \frac{m}{2} \cdot (z_2 + 2) \qquad (11)$$

Now one can determine the minimum pressure angle of the external gear (12, 13):

$$\begin{cases} tg\alpha_{1m} = \dfrac{N}{r_{b1}} \\ N = (r_{b1}+r_{b2})\cdot tg\alpha_0 - \sqrt{r_{a2}^2 - r_{b2}^2} = \dfrac{1}{2}\cdot m\cdot(z_1+z_2)\cdot\sin\alpha_0 - \dfrac{m}{2}\cdot\sqrt{(z_2+2)^2 - z_2^2\cdot\cos^2\alpha_0} = \\ = \dfrac{m}{2}\cdot[(z_1+z_2)\cdot\sin\alpha_0 - \sqrt{z_2^2\cdot\sin^2\alpha_0 + 4\cdot z_2 + 4}] \end{cases} \tag{12}$$

$$tg\alpha_{1m} = [(z_1+z_2)\cdot\sin\alpha_0 - \sqrt{z_2^2\cdot\sin^2\alpha_0 + 4\cdot z_2 + 4}]/(z_1\cdot\cos\alpha_0) \tag{13}$$

Now one can determine, for the external gear, the minimum (13) and the maximum (9) pressure angle for the right teeth. For the external gear with bended teeth ($\beta \neq 0$) one uses the relations (14, 15 and 16):

$$tg\alpha_t = \frac{tg\alpha_0}{\cos\beta} \tag{14}$$

$$tg\alpha_{1m} = [(z_1+z_2)\cdot\frac{\sin\alpha_t}{\cos\beta} - \sqrt{z_2^2\cdot\frac{\sin^2\alpha_t}{\cos^2\beta} + 4\cdot\frac{z_2}{\cos\beta} + 4}]\cdot\frac{\cos\beta}{z_1\cdot\cos\alpha_t} \tag{15}$$

$$\cos\alpha_{1M} = \frac{\dfrac{z_1\cdot\cos\alpha_t}{\cos\beta}}{\dfrac{z_1}{\cos\beta}+2} \tag{16}$$

For the internal gear with bended teeth ($\beta \neq 0$) one uses the relations (14 with 17, 18-A or with 19, 20-B):

A. When the driving wheel 1 has external teeth:

$$tg\alpha_{1m} = [(z_1-z_2)\cdot\frac{\sin\alpha_t}{\cos\beta} + \sqrt{z_2^2\cdot\frac{\sin^2\alpha_t}{\cos^2\beta} - 4\cdot\frac{z_2}{\cos\beta} + 4}]\cdot\frac{\cos\beta}{z_1\cdot\cos\alpha_t} \tag{17}$$

$$\cos\alpha_{1M} = \frac{\dfrac{z_1\cdot\cos\alpha_t}{\cos\beta}}{\dfrac{z_1}{\cos\beta}+2} \tag{18}$$

B. When the driving wheel 1 has internal teeth:

$$tg\alpha_{1M} = [(z_1-z_2)\cdot\frac{\sin\alpha_t}{\cos\beta} + \sqrt{z_2^2\cdot\frac{\sin^2\alpha_t}{\cos^2\beta} + 4\cdot\frac{z_2}{\cos\beta} + 4}]\cdot\frac{\cos\beta}{z_1\cdot\cos\alpha_t} \tag{19}$$

$$\cos\alpha_{1m} = \frac{\dfrac{z_1\cdot\cos\alpha_t}{\cos\beta}}{\dfrac{z_1}{\cos\beta}-2} \tag{20}$$

Determining the Efficiency

The efficiency of the gear will be calculated through the integration of momentary efficiency on all sections of gearing movement, namely from the minimum pressure angle to the maximum pressure angle, the relation (21), [2, 3]:

$$\eta = \frac{1}{\Delta\alpha} \cdot \int_{a_m}^{\alpha_M} \eta_i \cdot d\alpha = \frac{1}{\Delta\alpha} \int_{\alpha_m}^{\alpha_M} \cos^2\alpha \cdot d\alpha = \frac{1}{2\cdot\Delta\alpha} \cdot \left[\frac{1}{2} \cdot \sin(2\cdot\alpha) + \alpha\right]_{\alpha_m}^{\alpha_M} = \tag{21}$$

$$= \frac{1}{2\cdot\Delta\alpha}\left[\frac{\sin(2\alpha_M) - \sin(2\alpha_m)}{2} + \Delta\alpha\right] = \frac{\sin(2\cdot\alpha_M) - \sin(2\cdot\alpha_m)}{4\cdot(\alpha_M - \alpha_m)} + 0.5$$

More precise (5) for determining momentary efficiency is the relation (22):

$$\begin{cases} \eta_i = \cos^2\alpha_1 & (5) \\ \eta_i = \cos^2\alpha_1 \cdot \dfrac{1}{1 + \cos^2\alpha_1 \cdot \dfrac{\pi}{z_1} \cdot \left(\dfrac{\pi}{2\cdot z_1} + tg\alpha_1\right)} & (22) \end{cases}$$

Conclusion

The input parameters are: z_1 = the number of teeth for the driving wheel 1;
z_2 = the number of teeth for the driven wheel 2, or the ratio of transmission, i ($i_{12}=-z_2/z_1$);
α_0 = the pressure angle normal on the divided circle;
β = the bend angle.

Forest C

tity Gear

The efficiency of the gear increases when the number of teeth for the driving wheel 1, z_1, increases too, and when the pressure angle, α_0, diminishes. In addition, z_2 or i_{12} are not much influence on the efficiency value.

One can easily see that for the value $\alpha_0=20^\circ$, the efficiency takes roughly the value $\eta\approx0.89$ for any values of the others parameters (this justifies the choice of this value, $\alpha_0=20^\circ$, for the standard pressure angle of reference). The better efficiency may be obtained only for a $\alpha_0\neq20^\circ$, the efficiency takes roughly the value $\eta\approx0.89$ for any values of the other parameters (this justifies the choice of this value,α_0 =20°, for the standard pressure angle of reference). The better efficiency may be obtained only for a $\alpha_0\neq20°$. But the pressure angle of reference, α_0, can be decreased at the same time the number of teeth for the driving wheel 1, z_1, increases, to increase the gears' efficiency.

The module of the gear, m, has no influence on the gear's efficiency value. When α_0 is diminished one can take a higher normal module for increasing the addendum of teeth, but the increase of the m at the same time with the increase of the z_1 can lead to a greater gauge. The gears' efficiency, η, is really a function of α_0 and z_1: $\eta=f(\alpha_0,z_1)$; α_m and α_M are just the intermediate parameters. For good projection of the gear it's necessary to have a z_1 and z_2 greater than 30-60, but this condition may increase the gauge of mechanism.

In this paper we have discussed how one determines the dynamics efficiency, but at the transmission's gears the dynamics efficiency is the same as the mechanical efficiency. This is a greater advantage of the transmission's gears. This advantage of the gear's mechanisms may be found at the cam's mechanisms with plate followers, as well.

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