E(S)TIMAT √GEAR FATIGUE LIFE

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USING THE STRENGTH-LIFE THEORY CAN HELP AVOID GEAR FATIGUE FAILURE, AND THE RESULTING DISRUPTION OF WHOLE MANUFACTURING SYSTEMS.

By G. González Rey, R. J. García Martín, and P. Frechilla Fernández

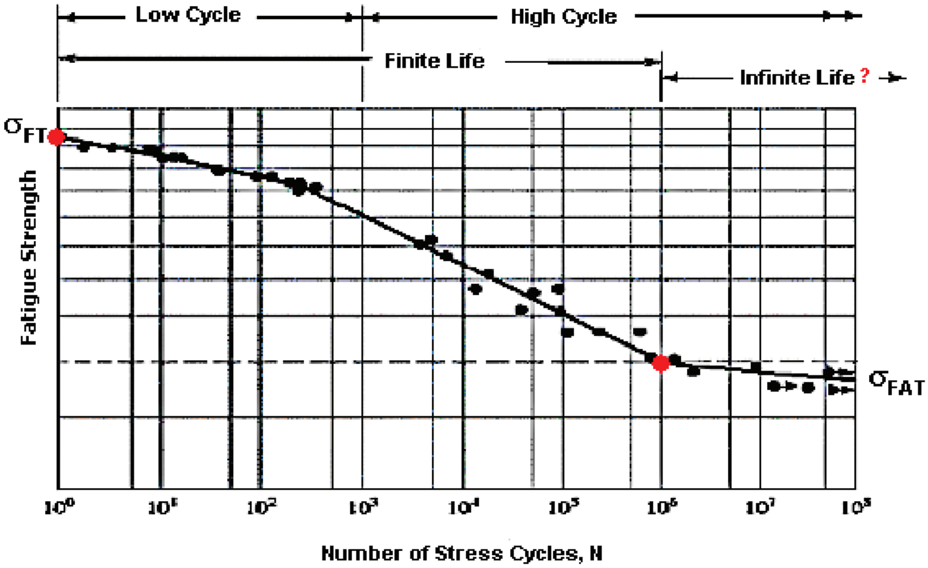

FIGURE 1: DIAGRAM OF WÖHLER WITH ACTUAL APPEARANCE OF GEAR STEEL BEHAVIOR. GRAPH PROVIDES THE CORRESPONDING FATIGUE STRENGTH FOR STEEL REPORTED AT A SPECIFIC NUMBER OF STRESS CYCLES.

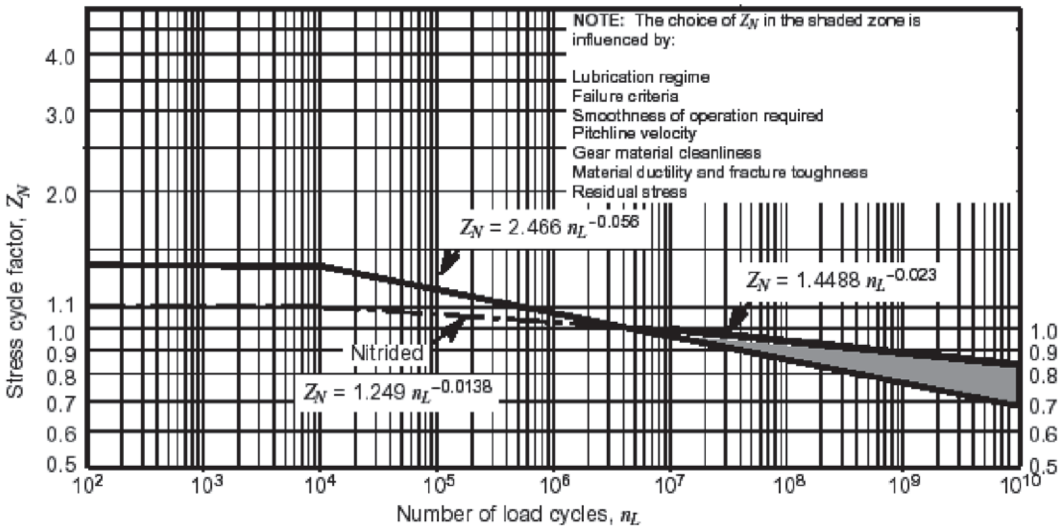

FIGURE 2: PITTING RESISTANCE STRESS CYCLES FACTOR, Z_N.[1]

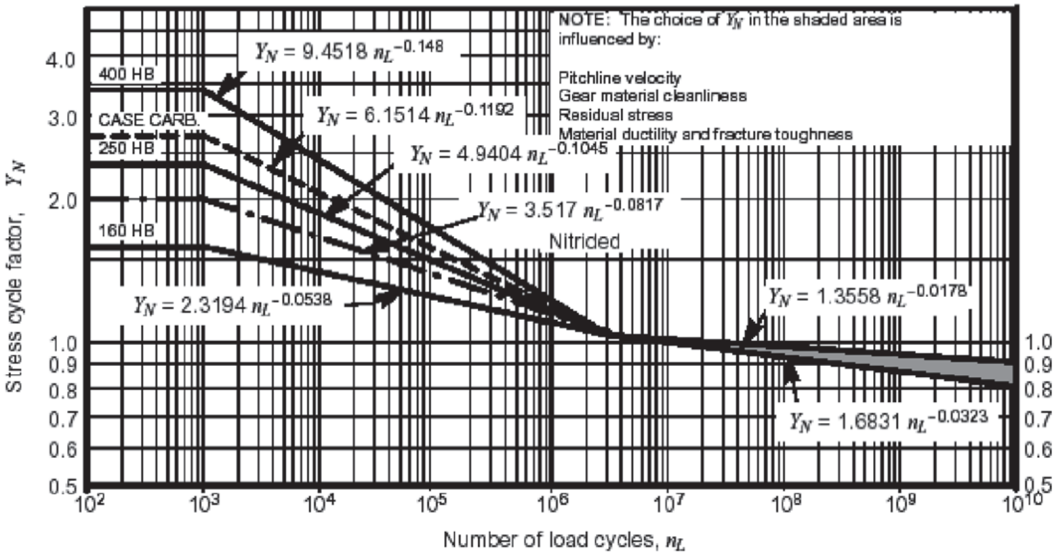

FIGURE 3: BENDING STRENGTH STRESS CYCLES FACTOR, Y_N.[1]

Fatigue failure of gears can lead to the catastrophic failure of equipment, taking into account that gears are important elements in the power transmission systems of many modern machines. Because of this, effective procedures and information to evaluate the load capacity and useful life of gears are needed by specialists in several fields of engineering application, including those involved with disaster preparedness and management in the fields of transportation, power generation, and the mechanical industry. The actual practice of engineering and increase of the work speeds in current applications of gears has required better specification of steel fatigue behavior for numbers of cycles greater than 10^6 or 10^7. In this sense, AGMA Standard 2105-D04 has introduced useful information to consider the fatigue load capacity of steel gears in the case of a high number of cycles. In this presentation, the procedure and formulas to estimate a value of gear life expectancy for a high number of cycles is given. The procedure takes into account the pitting resistance (surface fatigue failure) and bending strength capacity (volumetric fatigue failure) of spur and helical gears. Formulas are based on the AGMA Standard 2105-D04 for calculation of the load capacity of cylindrical gears.

The Stress-life Method

The distinguishing characteristic of materials associate with the lost of resistance under the action of repeated or fluctuating stresses is called fatigue failure. The study of fatigue failure is not an exact and absolute science, of which precise results can be obtained. The prediction of fatigue fracture is very often approximate and relative, with many components of the statistical calculation, and there are a great many factors to be considered, even for very simple load cases. In this sense the determination of the fatigue limit for materials with industrial purposes—in particular the steel—demands a great variety of test to define the magnitude of fatigue limit reported at a specific number of cycles.

In practice, gears are mostly operated under variable loads. Even in a continuous process the load acting on gear teeth is fluctuating due to the tooth contact process and operational conditions under which the gears shall perform. Under these variable loads a tooth breakage, which most often results in a total gear failure, must be take into account during the stages of gear design or load capacity calculation. This fact has demanded that new fatigue tests for gear materials be carried out and the fatigue resistance behavior with a high number of load cycles be analyzed.

As it is known, there are a great many factors to be considered during the study of fatigue phenomena. The methods of fatigue failure analysis are inexact and only approximate results can be obtained. Thus, more-exact methods require that more data be derived from practical testing and statistical calculation. A Whöler's, or strength-life (σ_{FAT} - N) diagram is the most widely used graph to provide the corresponding fatigue strength of a material reported at a specific number of stress cycles (see figure 1). In the Whöler diagram it is usual to represent the logarithm of the fatigue strength (Log σ_{FAT}) in the function of the logarithm of the number of cycles (Log N).

The fatigue failure analysis based on stress-life method is especially useful for a wide range of gear design applications and represents high-cycle applications adequately. In particular, the steel for gears requires a great variety of tests to define the fatigue strength versus the number of load cycles. In theory, it is often accepted that the line in the case of stress cycles greater than 10^6 or 10^7 cycles behaves with slope zero and failure will not occur, no matter how great the number of cycles. The stress value corresponding with the point of inflection in the graph is declared fatigue limit or endurance limit.

Figure 1 shows the actual appearance of gear steel behavior with a small and very significant modification: the graph becomes not totally horizontal after the steel has been stressed for a number of cycles greater than the basic number of cycles for established typical fatigue strength (N = 10^6 ...10^7). Moreover, it is possible to distinguish a significant change in the slope of the line near to 10^6 cycles. It is different than the classical infinite life appearance of steel behavior.

Gear performance demands load capacity for a number of stress cycles greater than the basic number of cycles for fatigue strength. In these situations it is useful to consider the fatigue resistance level in case of a high number of stress cycles.

The necessity for greater accuracy in the determination of fatigue limit for steel with applications in high speed gears has led to tests and new studies in the zone of a high stress cycle. AGMA Standard 2105-D04 is a good example of improvements and precision of the steel gear behavior. Formulas to evaluate the permissible strength for the volumetric and superficial fatigue of steel with application on cylindrical involute gears with external teeth gears are given on AGMA Standard 2105-D04 as follows.

$$\left|\sigma_F\right| = \frac{\sigma_{F\lim} \cdot Y_N}{S_F \cdot Y_\theta \cdot Y_Z} \tag{1}$$

$$\left|\sigma_H\right| = \frac{\sigma_{H\lim} \cdot Z_N \cdot Z_W}{S_H \cdot Y_\theta \cdot Y_Z} \tag{2}$$

$[\sigma_F]$: Permissible bending stress taking into account fatigue strength, [MPa].
$[\sigma_H]$: Permissible contact stress taking into account fatigue strength, [MPa].
σ_{Flim}: Fatigue limit for bending stress and unidirectional loading, [MPa].
σ_{Hlim}: Fatigue limit taking into account contact stress, [MPa].
S_F: Safety factor for bending strength.
S_H: Safety factor for pitting.
Y_N: Stress cycle factor for bending strength.
Z_N: stress cycle factor for pitting resistance.
Y_θ: Temperature factor.
Y_Z: Reliability factor.
Z_W: Hardness ratio factor for pitting resistance.

Particularly, the stress cycle factors take into account the strength-life characteristics of the gear material. Factors Z_N and Y_N, adjust the fatigue limit stress for the required number of cycles of operation as compared with fatigue limit stress established by testing at the basic number of cycles (N = 10^6 ...10^7 cycles). In the case of gears, the number of stress cycles is defined as the number of mesh contacts, under load, of the gear tooth being analyzed.

At the present time there is insufficient data to provide accurate stress cycle curves for all types of gears and gear applications. Experience, however, suggests that new stress cycle curves for pitting resistance and bending strength of steel gears as shown in AGMA Standard 2105-D04. Taking into account the current information about the behavior of the fatigue load capacity of steel for gears, it becomes clear how important it is to formulate a new direction and a method for estimating expected life in the case of a high number of cycles.

The purpose of this paper is to establish a procedure and formulas to estimate a value of gear expected life for a high number of cycles. The procedure takes into account the pitting resistance (surface fatigue failure) and bending strength capacity (volumetric fatigue failure) of spur and helical gears. The equations presented have been redefined according to the formulas for load capacity in AGMA Standard 2105-D04.

Determination of Stress Cycle Factors

Rating methods accepted by standards to evaluate the load capacity of external spur and helical involute gear teeth operating on parallel axes are based on the contact stress resistance and bending strength [1, 2, 3, 4]. The formulas evaluate gear tooth capacity as influenced by the major factors which affect progressive pitting of the teeth and gear tooth fracture at the fillet radius. The pitting and fracture of gear teeth are considered to be a fatigue phenomenon depending on stress cycles. Certification of gear load capacity is based on the confrontations of stress calculated by gear-tooth rating formulae with the bending and contact permissible stresses for gear materials.

The actual cylindrical gear-tooth rating formulae for pitting resistance are based on Hertz's results for the calculation of contact pressure between two curved surfaces. They have also been improved with modifications in the new standards to consider load sharing between adjacent teeth, the load increment due to external and internal dynamic loads, uneven distribution of load over the facewidth due to mesh misalignment caused by inaccuracies in manufacture, and elastic deformations, etc. The formulae for bending-strength rating are based on cantilever-projection theory. The maximum tensile stress at the tooth-root (in the direction of the tooth height) which may not exceed the permissible bending stress for the material is the basis for rating the bending strength of gear teeth. Just the same as in the calculation of tooth contact stress for pitting resistance, the calculating of tooth root strength takes into account load sharing between adjacent teeth, an increment of nominal load due to non-uniform distribution of load on the tooth face, and some external and internal dynamic load.

AGMA Standard 2105-D04 provides the following rating formulas and permissible stresses applicable for calculating the pitting resistance and bending strength of external cylindrical involute gear teeth operating on parallel axes.

$$\sigma_H = Z_E \cdot \sqrt{\frac{F_T \cdot K_O \cdot K_V \cdot K_H \cdot K_S \cdot Z_R}{b \cdot d_{w1} \cdot Z_I}} \le [\sigma_H] = \frac{\sigma_{H\lim} \cdot Z_N \cdot Z_W}{S_H \cdot Y_\theta \cdot Y_Z} \qquad (3)$$

$$\sigma_F = F_T \cdot K_O \cdot K_V \cdot K_H \cdot K_S \cdot \frac{K_B}{b \cdot m_T \cdot Y_J} \le [\sigma_F] = \frac{\sigma_{F\lim} \cdot Y_N}{S_F \cdot C_T \cdot Y_Z} \qquad (4)$$

Where:

σ_F: Bending tooth-root stress, [MPa].

σ_H: Contact tooth-flank stress, [MPa].

Z_E: Elastic coefficient, [Mpa1/2].

F_T: Transmitted tangential load, [N].

K_O: Overload factor.

K_V: Dynamic factor.

K_H: Load distribution factor.

K_S: Size factor.

Z_R: Surface condition factor for pitting resistance

K_B: Rim thickness factor.

b: Facewidth, [mm]

d_{w1}: Operating pitch diameter of pinion, [mm].

m_t: Transverse module, [mm]

Z_I: Geometry factor for pitting resistance.

Y_J: Geometry factor for bending strength.

By means of mathematical processing of formulas (3) and (4) it is possible to determine the stress cycle factors for pitting resistance and bending strength according to equations (5) and (6).

$$Z_N = Z_E \cdot \sqrt{\frac{F_T \cdot K_O \cdot K_V \cdot K_H \cdot K_S \cdot Z_R}{b \cdot d_{W1} \cdot Z_I}} \cdot \frac{S_H \cdot Y_\theta \cdot Y_Z}{\sigma_{H\lim} \cdot Z_W} \qquad (5)$$

$$Y_N = \frac{F_T \cdot K_O \cdot K_V \cdot K_H \cdot K_S \cdot K_B \cdot S_F \cdot Y_\theta \cdot Y_Z}{\sigma_{F\lim} \cdot b \cdot m_T \cdot Y_J} \qquad (6)$$

Determination of the Expected Fatigue Lifetime

Knowing the interrelation of factors Z_N and Y_N with the fatigue limit stress equivalent to a certain number of load cycles, it is possible to determine the useful expected fatigue lifetime in the condition of same bending and contact stresses in the teeth with corresponding permissible stresses for failure. Under these conditions, the number of load cycles expected by pitting (n_{Lh}) or fatigue fracture (n_{Lf}) can be evaluated with the stress cycle factors Z_N and Y_N determined by the formulas (5)-(6) and graphical information presented on AGMA 2105-D04 (see figures 2 and 3). Once certain that the numbers of load cycles corresponding to calculated values of factors Z_N and Y_N, the hours of expected fatigue lifetime (HσF and HσH) can be known by means of equations (7) and (8).

$$H_{\sigma_H} = \frac{n_{Lh}}{60 \cdot n \cdot q} \quad \text{[hours]} \qquad (7)$$

$$H_{\sigma_F} = \frac{n_{Lf}}{60 \cdot n \cdot q} \quad \text{[hours]} \qquad (8)$$

Where:

n_{Lh} : Number of load cycles expected by pitting in corresponding with stress cycle factors Z_N in figure 3.

n_{Lf} : Number of load cycles expected by fatigue fracture in corresponding with stress cycle factors Y_N in figure 4.

n : Rotational speed,(min-1)

q : Number of load application by 1 turn of gear. It can be different for bending stress or contact stress.

Sample Case

With the intention of demonstrating the procedure to estimate the useful expected fatigue lifetime of cylindrical gears, the calculation of the expected useful life of the pinion in a helical gear is presented (see tables 1 and 2). In particular, the gear transmission analyzed corresponds to the first stage of speed reducer applied in the gear transmission of a sugar cane mill. Field studies show gear failure by pitting after 10 years of sugar cane harvesting. It should be noted that the calculation of gear load capacity by pitting resistance was sufficient in case of classical theory of fatigue-life. The results that take into account the new fatigue resistance level with precision of stress cycle factors are more real (see table 2).

In general, safety factors must be established from a thorough analysis of the service experience with a particular application. A minimum safety factor is normally established for the designer by specific agreement between the manufacturer and purchaser. When specific service experience is not available, a thorough analytical investigation should be made. It is certain that

Description	Value
Number of teeth in pinion	$z_1 = 21$
Number of teeth in gear.	$z_2 = 44$
Operating pitch diameter of pinion	$d_{w1} = 135{,}7$ mm
Transverse module	$m_t = 6{,}46$
Facewidth	b = 52 mm
Pressure angle	$\alpha = 20°$
Addendum tooth factor	$h_a^* = 1$
Helix angle at standard pitch diameter	$\beta = 21{,}75°$
Geometry factor for pitting resistance	$Z_I = 0{,}181$
Transmitted tangential load	$F_t = 23000$ N
Rotational speed	$n = 1120\ \text{min}^{-1}$
Overload factor	$K_O = 1$
Dynamic factor.	$K_V = 1{,}28$
Load distribution factor	$K_H = 1{,}185$
Size factor	$K_s = 1{,}05$
Surface condition factor for pitting resistance	$Z_R = 1$
Fatigue limit taking into account contact stress	$\sigma_{H\lim} = 1345$ MPa
Temperature factor	$Y_\theta = 1$ (T < 100°C)
Hardness ratio factor for pitting resistance	Zw = 1
Reliability factor	Yz = 1 (99%)
Safety factor for pitting.	$S_H = 1$

TABLE 1: INITIAL DATA TO ESTIMATE THE USEFUL EXPECTED FATIGUE LIFETIME OF CYLINDRICAL GEARS CONSIDERING PITTING RESISTANCE OF THE TEETH.

	Formulas	Results
1	$Z_N = Z_E \cdot \sqrt{\frac{F_T \cdot K_A \cdot K_V \cdot K_\beta \cdot Y_X}{b_W \cdot d_1 \cdot I}} \cdot \frac{S_H \cdot C_T \cdot Y_Z}{\sigma_{H\lim} \cdot Z_W}$ $Z_N = 190 \cdot \sqrt{\frac{23000 \cdot 1 \cdot 1{,}28 \cdot 1{,}185 \cdot 1{,}05}{52 \cdot 135{,}7 \cdot 0{,}181}} \cdot \frac{1 \cdot 1 \cdot 1}{1345 \cdot 1}$	$Z_N = 0{,}756$ Calculation in zone of high cycles.
2	By means of figure 3: $n_{Lh} = \sqrt[0{,}056]{\frac{2{,}466}{Z_N}}$ $n_{Lh} = \sqrt[0{,}056]{\frac{2{,}466}{0{,}756}}$	$n_{Lh} = 14{,}76 \times 10^8$ cycles
3	$H_{\sigma_H} = \frac{n_{Lh}}{60 \cdot n \cdot q}$ $H_{\sigma_H} = \frac{14{,}76 \times 10^8}{60 \cdot 1120 \cdot 1}$	$H_{\sigma_H} = 21966$ hours
4	Estimating life for daily work time of 8 hours	T = 2746 days ≈ 7,6 years
5	Estimating life for sugar harvest of 2160 hours	T = 10,2 years

TABLE 2: STRESS CYCLE FACTORS, NUMBER OF LOAD CYCLES EXPECTED BY PITTING AND LIFETIME.

TABLE 4: RECOMMENDED VALUES FOR THE MINIMUM SAFETY FACTORS IN SPECIFIC CASES.

Pitting	S_{Hmin}
Main recommendation value	1,3
Guaranteed high quality with homogeneous material structure	1,1
Guaranteed high quality with surface hardening	1,2
Dangerous consequences with homogeneous material structure	1,3
Dangerous consequences with surface hardening	1,4
When pitting is not dangerous	≥ 1
Breakage, fatigue load	S_{Fmin}
Main recommendation value	1,7
Guaranteed high quality	1,6
Roll material	1,9
Cast gears or work in high temperature	2,2

Requirements of application	Y_Z
Fewer than one failure in 10 000	1.50
Fewer than one failure in 1000	1.25
Fewer than one failure in 100	1.00

TABLE 3: RELIABILITY FACTORS, Y_Z[1]

the magnitude of safety and reliability factors can condition the value of estimating life, for good designs with proven values of safety and reliability are important (see tables 3-4).

Conclusions

An effective procedure, formulas, and information to estimate a value of expected fatigue life in the case of a steel cylindrical gear with a high number of cycles has been given. Formulas are based in the AGMA Standard 2105-D04 for calculation of the load capacity of cylindrical gears.

In this paper the stress cycle factors take into account the strength-life characteristics of the gear material, and it used the factors Z_N and Y_N to adjust the fatigue limit stress for the required number of cycles of operation. The procedure is fixed taking into account the pitting resistance and bending strength capacity of spur and helical gears.

Knowing the interrelation of factors Z_N and Y_N with the fatigue limit stress equivalent to a certain number of load cycles, it is possible to determine the useful expected fatigue lifetime in the condition of the same bending and contact stresses in the teeth with corresponding permissible stresses for failure. Under these conditions the number of load cycles expected by pitting (n_{Lh}) or fatigue fracture (n_{Lf}) can be evaluated with the stress cycle factors Z_N and Y_N determined by the formulas (5)-(6) and the graphical information presented on AGMA 2101-D04 (see figures 2 and 3). Once certain that the numbers of load cycles correspond to calculated values of factors Z_N and Y_N , the hours of expected fatigue lifetime ($H\sigma_F$ and $H\sigma_H$) can be determined by means of equations (7) and (8).

Some results of field studies show a good approximation between data from the field and the values obtained by means of the procedure described in this paper, but it is necessary to conduct more testing and data application to improve the results due to the great many factors to be considered in fatigue failure.

REFERENCES:

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ABOUT THE AUTHORS:

Dr. Gonzalo González Rey is principal professor in the machine elements division of the faculty of Mechanical Engineering at the Instituto Superior Politécnico José A. Echeverría (CUJAE) in Havana, Cuba. He is also an AGMA member with expertise in the area of ISO/TC60/WG6-13. He can be reached at cidim@mecanica.cujae.edu.cu. Eng. D. Roberto José García Martín is a collaborator professor of mechanical engineering at University of Salamanca-Spain at the campus of E.S.P. of Zamora. His background is in the area of design and machine control, and he can be reached at toles@usal.es. Prof. Pablo Frechilla Fernández is titular professor of mechanical engineering at the Salamanca University in Spain. His background is in machine design, with more than 30 years of experience as an advisor. He can be reached at pf2@usal.es.