Exhaustive research supports new methods for designing and manufacturing straight bevel gears that are stronger and more cost-effective than ever before.

an Optimized Approach to Straight Bevel Gear Design

By Dr. Gonzalo González Rey

The optimal—or "rational"—design of gears can be a challenge due to the high number of variables, limitations, and interrelationships involved. And also because, in each separate case, the objective functions, restrictions, and independent variables are generally different. Many of the specialists who have approached gear optimization by means of exhaustive search methods [Dana (1969), Escanaverino (1984), and Lopez (1993)] first organized an evaluation of the objective function with reasonable design parameter values in place, and then compared and retained the most extreme results of the function in each step.

In this particular case a functional methodology is given, with the objective being to establish general principles and procedures for the rational calculation of gears, with an emphasis on straight bevel gears. The procedure utilized has as its starting point a mathematical model based on the ISO standards for gear load capacity and a system generated with interrelation between gear geometry and resistance. The ISO standards for the load capacity calculation of bevel gears (ISO 10300-1,2,3) was used to develop the relationships between the geometry and tooth resistance. Additionally, the procedure includes the verification of the gear's geometric qualities by means of "indicators" to establish restrictions in the field of possible solutions. Some of them are:

- Transverse contact ratio ($\varepsilon_{v\alpha}$) greater than a minimum value
- Tooth topland thickness (s_a) greater than a minimum value
- Tooth interference occurrence is inadmissible
- Dimension of the outer tip diameter on gear (d^{ae2}) smaller than a maximum value

Maximum Torque on a Pinion Based in ISO Formulae for Verification of the Surface Load Capacity

The actual bevel gear-tooth rating formulae for pitting resistance are based, firstly, on Hertz's results for the calculation of contact pressure between two curved surfaces. They have also been improved with modifications in the ISO standards to consider load sharing between adjacent teeth, the position of the center of pressure on the tooth, the shape of the instantaneous area of contact, and the load concentration resulting from manufacturing uncertainties. Taking all of this into account, the ISO formulae for the calculation of surface durability of bevel gears (ISO 10300-2 and ISO 6336-5) were the basis to determine the objective function for evaluation of the maximum torque on a pinion according to the higher load capacity.

$$[T_{1C}] = 2{,}1664 \cdot 10^{-5} \cdot \frac{u \cdot l_b}{K_A \cdot K_V \cdot K_{H\beta} \cdot K_{H\propto} \cdot \sqrt{u2+1}} \cdot \left(\frac{\sigma_{HP} \cdot d_{m1}}{Z_{M\text{-}B} \cdot Z_H}\right)^2 \text{ (Nmm)}$$

Where:

K_A	Load application factor
K_V	Dynamic factor
$K_{H\beta}$	Face load factor for contact stress
$K_{H\alpha}$	Transverse load factor for contact stress
d_{m1}	Mean reference diameter on pinion (mm)
$Z_{M\text{-}B}$	Mid-zone factor
Z_H	Zone factor
u	Gear ration
l_b	Length of the line of contact (mm)
σ_{HP}	Permissible contact stress (MPa)
$[T_{1C}]$	Permissible (maximum) torque on pinion according to contact stress (Nmm)

Maximum Torque on a Pinion Based in ISO Formulae for Verification of the Tooth Root Strength

The actual bevel gear formulae for bending-strength rating are based on cantilever-projection theory and can be used for determining a load rating that will prevent tooth root fillet fracture (bending fatigue) during the design life of the gear teeth. In most cases, the maximum tensile stress at the tooth root (arising from bending at the root when the load is applied to the tooth flank) can be used as the criterion for the assessment of the bending tooth root strength, as when the allowable stress number is exceeded the teeth can experience breakage. When calculating the tooth root stresses of straight bevel gears, it is considered that the load is applied at the tooth tip of the virtual cylindrical gear, and that it is subsequently converted to the outer point of single-tooth contact with the aid of the contact-ratio factor $Y\varepsilon$. Just the same as in the calculation of tooth contact stress for pitting resistance, the calculating of tooth root strength takes into account load sharing between adjacent teeth, an increment of nominal load due to non-uniform distribution of load on the tooth face, and some external and internal dynamic load resulting from manufacturing uncertainties and tooth elastic deformation.

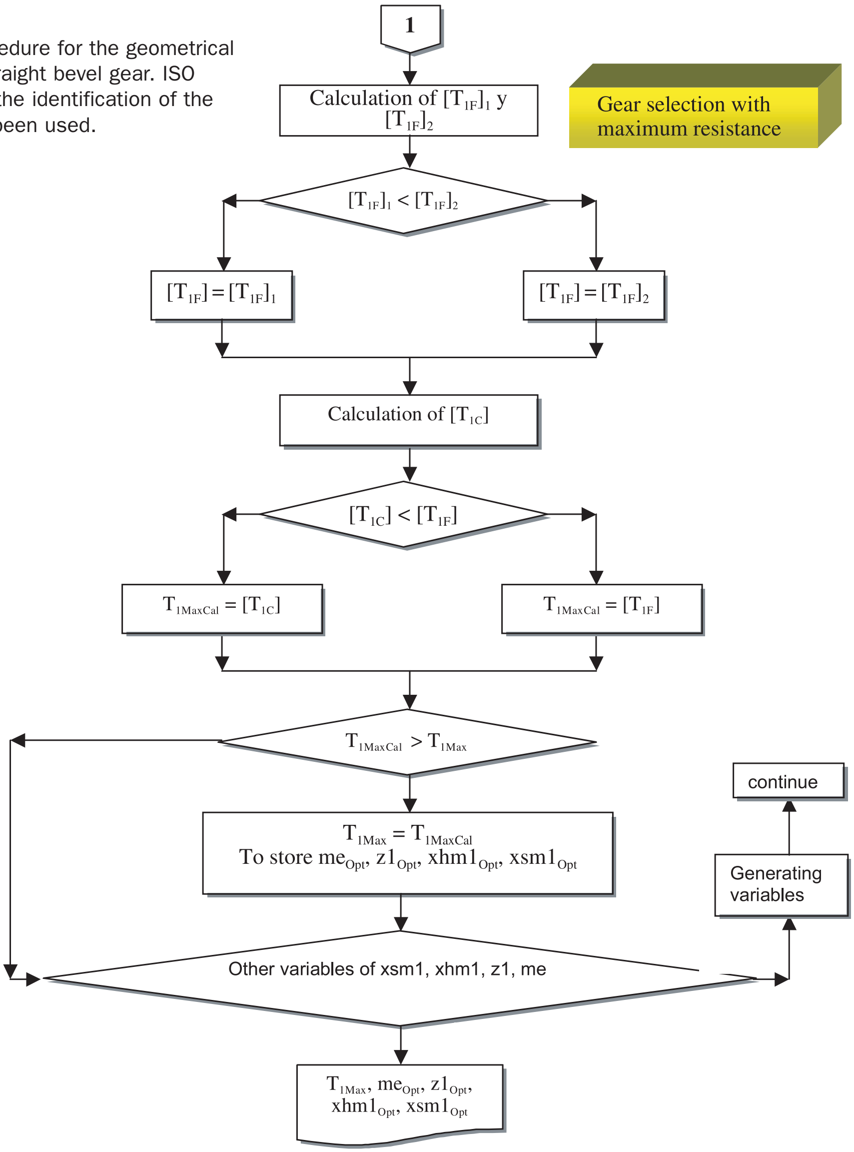
Taking this state of the art into account, the ISO formulae for the calculation of tooth root strength of bevel gears (ISO10300-3 and ISO 6336-5) were the basis to determine the other objective function for evaluation of the maximum torque on a pinion according with the higher load capacity.

$$[T_{1F}] = \frac{\sigma_{Fp} \cdot b \cdot m^2 \cdot z_1 \cdot Y_x \cdot Y_{\delta reIT}}{2 \cdot K_A \cdot K_V \cdot K_{F\beta} \cdot K_{F\propto} \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_\varepsilon \cdot Y_K} \text{ (Nmm)}$$

Where:

b	Face width (mm)
z_1	Number of teeth on pinion
m	Outer normal module (cutter module) (mm)
Y_X	Size factor for tooth root stress
$Y_{\delta relT}$	Relative sensitivity factor
$K_{F\beta}$	Face load factor for bending stress
$K_{F\alpha}$	Transverse load factor for bending stress
Y_{Fa}	Tooth form factor for load application at tip
Y_{Sa}	Stress correction factor for load application at tooth tip
Y_ε	Contact ratio factor for tooth root stress

Figure 1 — Procedure for the geometrical synthesis of a straight bevel gear. ISO nomenclature in the identification of the parameters has been used.



Y_K	Bevel gear factor
σ_{FP}	Permissible tooth root stress (MPa)
$[T_{1F}]$	Permissible (maximum) torque on pinion according to tooth root strength (Nmm)

A Rational Calculation of Straight Bevel Gear with High Load Capacity for the Condition of Tooth Pitting and Bending Fatigue

In the proposed method, the rational parameters of the straight bevel geometry are obtained by means of an organized evaluation of the two objective functions that allows, in each step, to compare and to retain the extreme value in the functions (instantaneous optimum) and all values of each design parameter involved. The design parameters obtained by these means belong to the group of acceptable values and possible solutions in the design problem. In this case, the variable to maximize was the pinion torque applied in the gear transmission with restrictions of:

- Maximum load capacity to contact and bending stresses
- Transverse contact ratio greater than a minimum value
- Not tooth interference occurrence
- Tooth topland thickness greater than a minimum value
- Outer tip diameter on gear no greater than a maximum value

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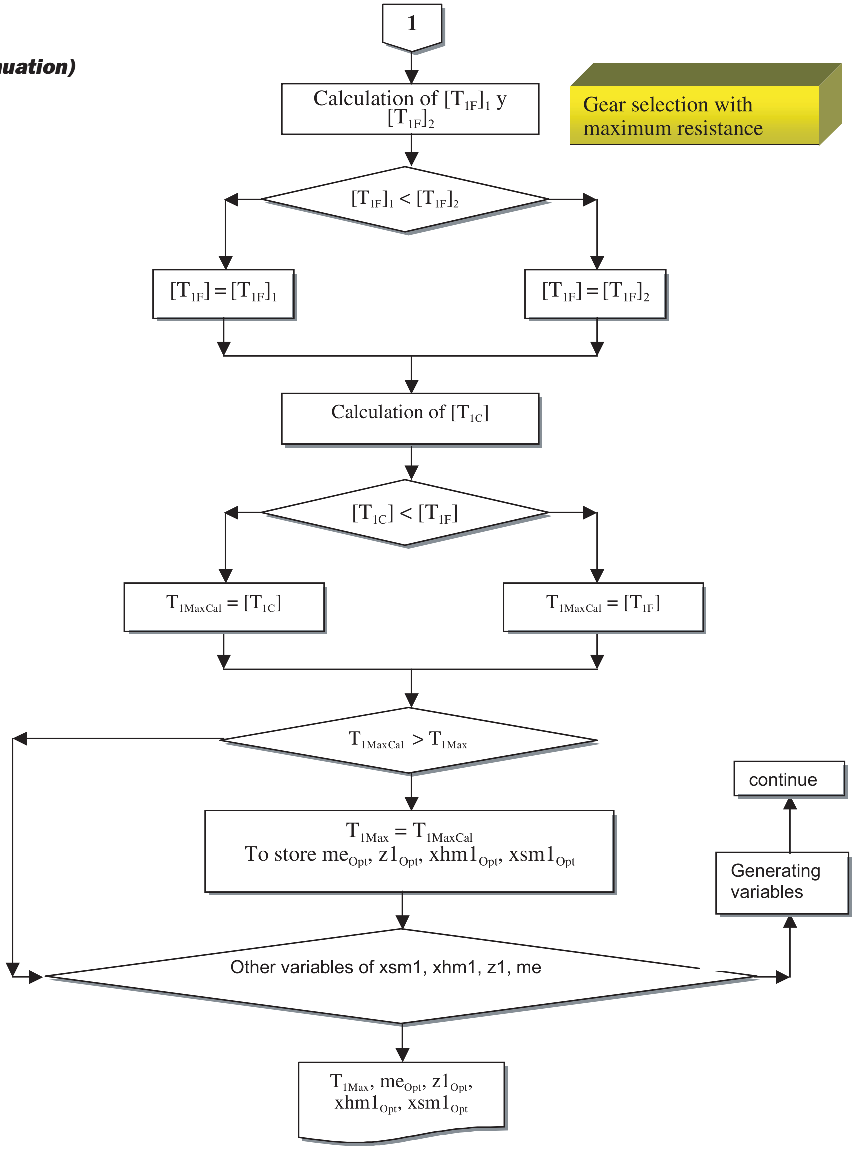
Figure 1 (continuation)



Due to space limitations, a simplified algorithm of the calculation procedure of the rational geometry for a straight bevel gear is shown in Figure 1.

Results Obtained Using the Proposed Calculation Procedure

The described procedure was the base for two computation programs in Visual Basic 4.0. The results obtained in the various running of computation programs showed the possibilities of these procedures for the analysis of new bevel gear designs.

The validation of rational design results was achieved by taking 98 straight bevel gears referenced in technical catalogues and literature by American companies such as The Gleason Works, Browning Manufacturing, and Martin Sprocket & Gears. Results for two groups of designs are presented in Figure 2.

It can be noted that in Figure 2 the load capacities calculated according to ISO 10300 were superior in an average of 9 percent on straight bevel gears whose geometry was obtained using the procedures exposed. Taking into account all straight bevel gears in the validation, the ISO load capacity was greater than an average of 15 percent.

The results reflected in Figure 3 are corresponding with a rational geometric synthesis of straight bevel gears at 90°

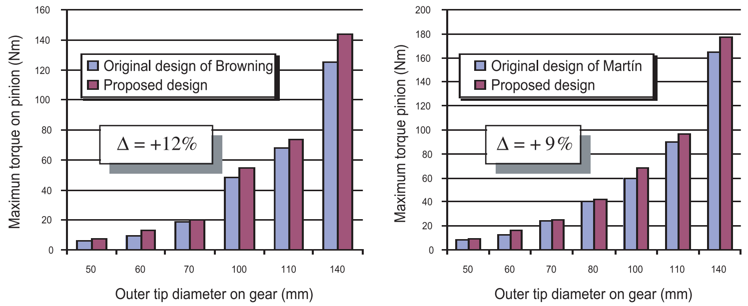


Figure 2 — Comparison between some straight bevel gear capacities calculated using the geometry declared by the manufacturers and straight bevel gears with geometry determined by means of the procedure described in this work

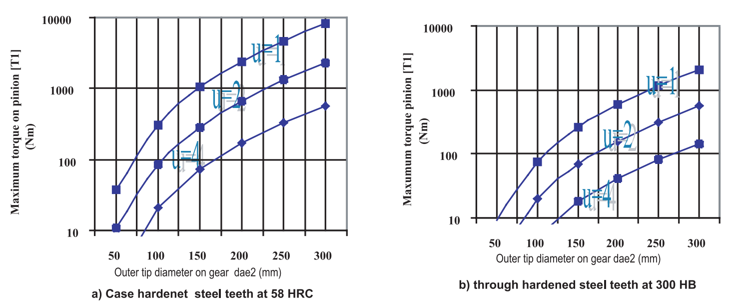


Figure 3 — Maximum ISO load capacities in straight bevel gear at 90° shaft angle with case hardened steel teeth. Conditions: crowning teeth, pinion in cantilever mounted, outer modules according to ISO standards, it was assigned the value 1 to the safety coefficients for contact and flexion stresses, material quality grade MQ, ISO quality $Q_{ISO} = 8^{vo}$, standard cutting tool parameters (a = 20°, ha* = 1.0, c* = 0.25, ρ_f* = 0,25).

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shaft angle, with maximum ISO load capacity and restrictions of the outer tip diameter of the gear. Due to space restrictions, only some of the results obtained are presented here.

"Based on the new standard ISO 10300, this procedure is conservatively considered to provide economic savings of 10 percent, and it demonstrated possibilities to increase the load capacity up to 15 percent."

Conclusions

A procedure was developed for the rational calculation of straight bevel gears at 90°. This procedure was based on the new standard ISO 10300. It is conservatively considered to provide economic savings of 10 percent, taking into account a more-compact design with higher load capacity. The results achieved using the procedures given were compared with successful straight bevel gears, and they demonstrated possibilities to increase the load capacity up to 15 percent. Moreover, the results have shown that the exhaustive search technique is very effective in the optimized calculation of gears. The results presented in Figure 3 can be used to obtain a rational design according to the declared conditions.

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