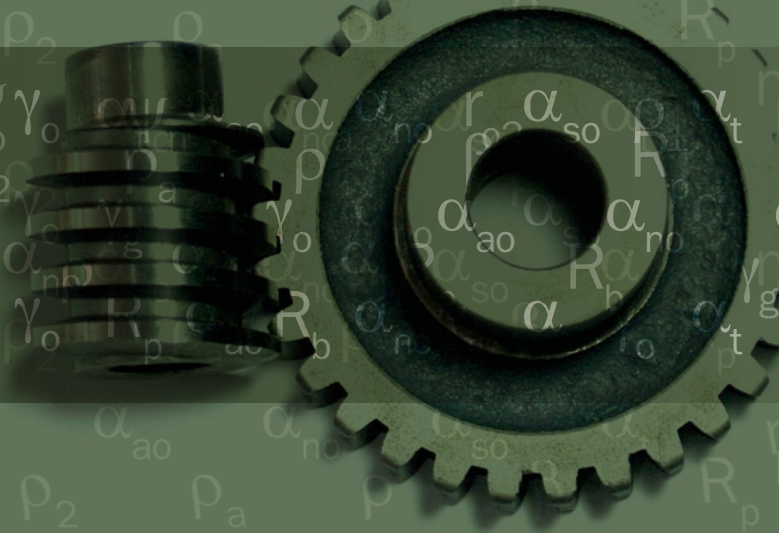


RELATIVE CURVATURE of *Worm & Wheel* with STRAIGHT LINE GENERATRIX

By P.K. Banerjee



You're sure to benefit from reading this fascinating study presented by a gearbox specialist with Tata Steel on calculating—and better understanding—worm/wheel contact.

Nomenclature:

m_a	=	Axial module (mm)
Ψ	=	Helix angle (deg.)
ψ_n	=	Helix angle at normal plane (deg.)
α_a	=	Axial pressure angle (deg.)
r_{o2}	=	Pitch circle radius of wheel (mm)
ρ_1, ρ_2	=	Curvatures for worm and wheel respectively. (mm-1)
ρ_a	=	Equivalent curvature at axial plane. (mm-1)
ρ	=	Equivalent curvature at normal plane.(mm-1)
β	=	Angle (polar coordinate) with respect to the vertical plane for point of contact P
R_p	=	Pitch circle radius. (mm)
R_b	=	Base circle radius. (mm)
γ_g	=	Lead angle at base circle diameter.(deg.)
γ_o	=	Lead angle at pitch circle diameter.(deg.)
α_{ao}	=	Axial pressure angle at pitch circle diameter.
α_{no}	=	Normal pressure angle at pitch circle diameter. (deg.)
α_{so}	=	Transverse pressure angle at pitch circle diameter. (deg.)
α_t	=	Angle of generatrix (or pr. Angle of straight tooth rack). (deg.)

The contact on single enveloping worm gear drives is the line contact. The changing form of the worm across the face of the gear makes it impossible to derive a simple mathematical expression for the contact curvature. In addition, the combination of the lead angle of the worm and position of the pitch plane of the worm in reference to the worm thread profile has predominant influence on the position and form of the actual contact line. The author here presents a report on the relative curvature of the worm and wheel at sections parallel to the axial section of a worm.

Curvature & Radius of Curvature

A curve is the locus of a point whose position vector r relative to a fixed origin may be expressed as a function of a single variable parameter. It is usually convenient to choose as the scalar parameter the length s of the arc of the curve measured from a fixed point A. Let P, Q be the points on the curve whose position vectors are $r, r + \delta r$ corresponding to values $s, s + \delta s$ of the parameter then δr is the vector PQ. The quotient $(\delta r / \delta s)$ is a vector in the same direction as δr and in the limit, as δs tends to zero, this direction becomes that of the tangent at P. Moreover, the ratio of the lengths of chord PQ and the arc PQ tends to unify as Q moves up to coincide with P. Therefore, the limiting value of $(\delta r / \delta s)$ is a unit vector parallel to tangent to the curve at P and in the positive direction (see fig. 1). If it is designated by t , then:

$$t = \lim_{\delta s \rightarrow 0} (\delta r / \delta s)$$



The curvature of the curve at any point is the arc rate of rotation of the tangent. Thus, if $\delta\theta$ is the angle between tangents at P & Q, $\delta\theta / \delta s$ is the average curvature of the arc PQ, and its limiting value as δs tends to zero is the curvature at the point P. This is sometimes called the first curvature of circular curvature. We shall denote it by κ .

$$\text{Thus, } \kappa = \lim_{\delta s \rightarrow 0} (\delta\theta / \delta s) = d\theta / ds = \theta' = 1/\rho$$

ρ is called radius of circular curvature. The relative curvature of the surfaces in contact will be

$$1/\rho = 1/\rho_1 + 1/\rho_2$$

A negative sign will be attached with the radius (ρ_1 or ρ_2) which is concave.

Tooth Profiles & Geometrical Relations

For general industrial purposes, helicoid sections for worm and wheel are mostly involute helicoids and screw helicoids. Both of these helicoids can be generated with a straight line generatrix, therefore their tooth geometry is simpler. Involute helicoids may be generated either by hobbing or by lathe (see fig. 2). When it is generated by lathe, half of the cutter included angle must be equal to the lead angle of base helix. From the properties of the involute helicoids, it is known that the teeth flank will have some curvature in its transverse section and the radius of curvature at pitch point will be given by:

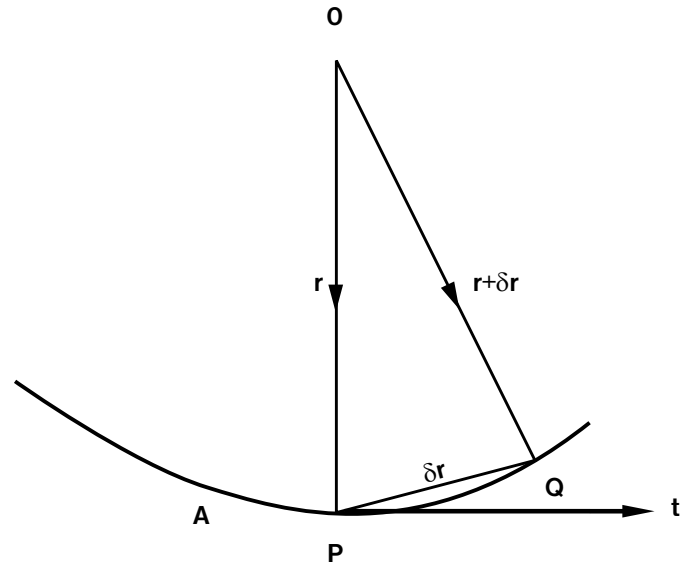


Fig. 1



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$$\rho = R_p \sin \alpha_s \quad (1)$$

A normal section of the curvature will have a radius of curvature:

$$\rho_n = R_p (\sin \psi_n \div \cos^2 \psi) = R_p (\sin \alpha_n / \cos^2 (90 - \gamma)) = R_p (\sin \alpha_n \div \sin^2 \gamma) \quad (2)$$

The radius of curvature at axial section is related with the radius of curvature at normal section according to the following equation:

$$1/\rho_A = (1/\cos \gamma) * (\cos \alpha_a / \cos \alpha_n)^3 [(1/\rho_n) + (\sin^2 \gamma / R_p) * \sin \alpha_n * (1 + \cos^2 \alpha_n)] \quad (3)$$

By putting the value of ρ_n from equation (2) in the equation (3) and simplifying it, the following equation is obtained:

$$1/\rho_A = (\sin^2 \gamma / R_p) * [(1/\sin^2 \alpha_n) + \sin \alpha_n * (1 + \cos^2 \alpha_n)] * (1/\cos \gamma) * (\cos \alpha_a / \cos \alpha_n)^3 \quad (4)$$

The radius of curvature at the axial section of the involute worm is obtained. The radius of curvature at axial section of screw helicoids is independent of its axial pressure angles, lead angle, and pitch diameter of the worm or wheel. Here curvature is a constant and is equal to zero.

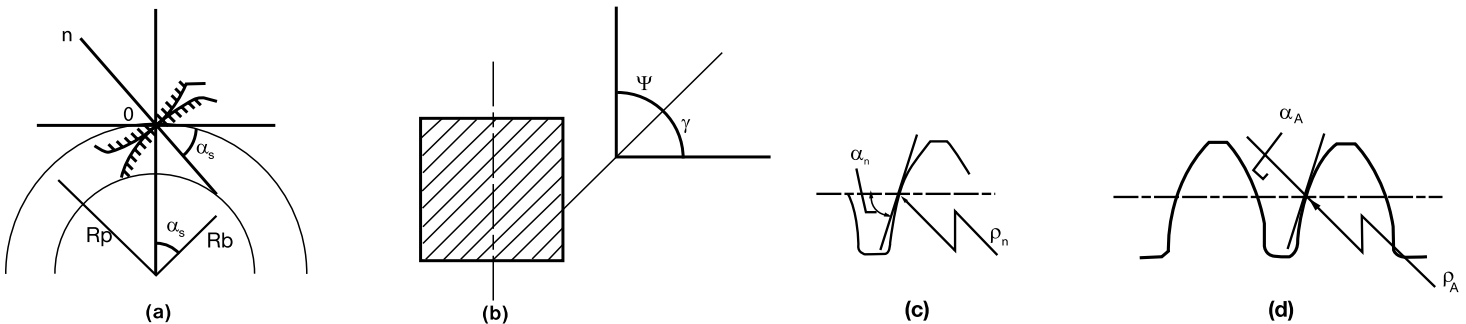


Fig. 2

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Curvature Radius at a Section Parallel to Axial Section

The radius of curvature of the worm tooth flank at a point P is to be determined [2]. We follow Cartesian coordinate system, in which the central point of the transverse section of the worm is the origin (see fig. 3).

The worm axis represents Z axis, and the X & Y axis are as shown in the transverse section of worm. The coordinate of any point P on the helicoids (flank) will be the function of x, y & z. But for any section parallel to axial section, the value of y will be constant. Therefore, for a particular point P, z is the function of x only. From the definition of the pressure angle at any section it is known that:

$$\tan \alpha = \partial Z / \partial X$$

$$\text{Or } (1 / \cos^2 \alpha) = 1 + (\partial Z / \partial X)^2$$

The point P in xx section may be indicated either by Cartesian coordinate system x, y, and z or by the polar coordinate β , r, z, so it is given that:

$$r^2 = x^2 + y^2 \quad \text{and} \quad \tan \beta = y/x$$

In the worm axial section AA, $Y_a = 0$, $X_a = r$ and z_a being a function of r is given by $z_a = f(r)$

$$\text{Therefore, } \tan \alpha_a = (\partial f(r) / \partial r) ; \quad 1 / \rho_a = -\cos^3 \alpha_a = (\partial^2 f(r) / \partial r^2)$$

The equation to the helicoids plane is:

$$z = f(r) - (\beta L / 2\pi)$$

$$\begin{aligned} \text{Therefore, in section xx, } \tan \alpha &= (\partial z / \partial x) \\ &= (\partial f(r) / \partial r)(\partial r / \partial x) - (\partial \beta / \partial x)(L / 2\pi) \\ &= (\partial f(r) / \partial r)(x/r) - (L / 2\pi)(y / (x^2 + y^2)) \\ &= \tan \alpha_a \cos \beta + \tan \gamma \sin \beta \end{aligned} \quad (5)$$

$$\begin{aligned} 1/\rho_1 &= -[\cos^3 \alpha (\partial z / \partial x)] \\ &= -\cos^3 \alpha [(\partial^2 f(r) / \partial r^2)(\partial r / \partial x)^2 + (\partial f(r) / \partial r)(\partial^2 r / \partial x^2) - (\partial^2 \beta / \partial x^2)(L / 2\pi)] \\ &= +\cos^3 \alpha [(\cos^3 \beta / (\rho_a \cos^3 \alpha_a)) - (\tan \alpha_a \sin^2 \beta / r) + (2 \tan \gamma \sin \beta \cos \beta / r)] \end{aligned} \quad (6)$$

Relative Curvature of the Worm and Wheel Teeth Profile

$$1/\rho = (r_{o2} \sin \alpha - (a^2 / \rho_1) + a) / (1 - (a / \rho_1))^2 \quad (7)$$

$$\text{When, } a = ((Rp - r \cos \beta) / \cos \alpha) \quad (8)$$

It is to be noted that for the present coordinate system ρ_2 is positive when wheel flank is convex. ρ_1 is positive when worm flank is concave [2].

EXAMPLE 1

To calculate the relative curvature of an involute helicoid worm at a section which is parallel to axial section, the point of contact is assumed to make an angle approximately 20 degree with vertical plane. The worm have following gear parameters:

(Axial) Module $m_a=5$, No. of start of worm $Z_1 = 1$ diametrical quotient $q = 10$. Root diameter $d_{\text{root}} = 38.20$ mm. Pitch circle diameter $d_o = 50$ mm. Addendum diameter $d_a = 60$ mm, lead angle (at pitch circle diameter) $\gamma_o = 5.71059^\circ$. Normal pressure angle (at pitch circle diameter) $\alpha_n = 20^\circ$. Working center distance $a' = 125$ mm. Wheel teeth number $Z_2 = 40$. Transmission ratio $i = 40$.

Solution: The exact coordinate point of contact in this case will be $\beta = 20.762$, $r = 26.28$ mm. (see ref. 3). The geometrical relations for involute helicoids worm gear is given as:

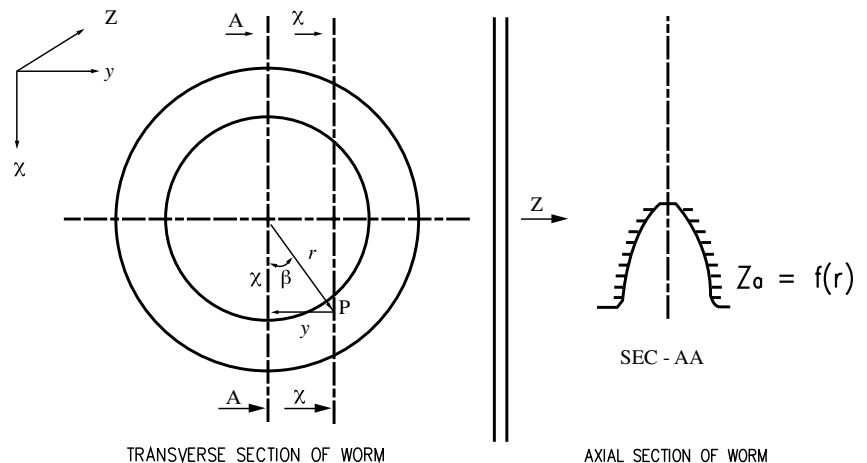


Fig. 3

$$\tan^2 \gamma_g = \tan^2 \alpha_t = \tan^2 \alpha_{ao} + \tan^2 \gamma_o \dots (i)$$

$$\tan \alpha_{ao} = \tan \alpha_{no} / \cos \gamma_o \dots (ii)$$

From equation (ii) $\alpha_{ao} = 20.09179$ degree. Here, $R_p = 25$ mm

From equation (i) $\alpha_t = 20.767152$ degree

Substituting the values in equation (4) $\rho_a = 273.80396$ mm

From equation (5)

$$\begin{aligned} \tan \alpha &= \tan 20.09179 \cos 20.762 \div \tan 5.71059 \sin 20.762 \\ &= 0.3774808 \\ \alpha &= 20.6806^\circ \end{aligned}$$

From equation (6) $\rho_1 = 278.91121$ mm

From equation (8) $a = 0.4559783$ mm

From equation (7) $\rho = 35.88828$ mm

EXAMPLE 2

To calculate the relative curvature of a screw helicoids worm at a section which is parallel to axial section. The point of contact is assumed to make an angle 20 degree with vertical plane. The worm of wheel have the same gear parameters as in example 1 except instead of normal pressure angle, axial pressure angle

(at p.c.d.) = 20 degree.

Solution: The contact position remains almost same for all types of worm and wheel which has got straight line generatrix (see note * at end). Therefore, here also it may be taken as:

$$\beta = 20.26 \text{ degree, } r = 26.28 \text{ mm}$$

At any position in axial section of screw helicoids $1/\rho_a = 0$

$$\text{From equation (6) } 1/\rho_1 = 0.00070569, \rho_1 = 1417.05 \text{ mm}$$

$$a = 0.4559783 \text{ mm}$$

From equation (7) $\rho = 35.79487$ mm

EXAMPLE 3

For the gear pair of example 1 calculate the relative curvature at a section parallel to vertical plane making an angle approximately equal to 30 degrees.

Solution: The exact point of contact in this case will be at $\beta = -28.155$ degree, $r = 28.35$. The negative sign attached with indicates angle is at opposite side of assumed direction. So it can be dropped.

$$\text{From equation (4) } 1/\rho_a = 0.00365224, \rho_a = 273.80396 \text{ mm}$$

From equation (5) $\alpha = 20.2888$ degree

$$\text{From equation (6) } 1/\rho_1 = 0.00325236, \rho_1 = 307.46823 \text{ mm}$$

$$a = 0.00484252 \text{ mm.}$$

From equation (7) $\rho = 34.679834$ mm.

EXAMPLE 4

For the gear pair of example 2 calculate relative curvature at a section parallel to vertical plane, making an angle of 30 degrees.

$$\text{Solution: } \beta = 28.1^\circ, r \approx 28.3 \text{ mm} \quad \alpha = 20.2888 \text{ degree}$$

$$\text{From equation (6) } 1/\rho = 0.000068584, \rho = 14580.64 \text{ mm}$$

$$\text{From equation (7) } \rho = 34.665665 \text{ mm.}$$

EXAMPLE 5

For the gear pair of example 1 calculate relative curvature exactly at axial section.

$$\text{Solution : Calculated } \rho_a = 273.80396 \text{ mm, } \beta = 0 \text{ degree,}$$

$$r = 25 \text{ mm} = R_p$$

$$\alpha = \alpha_a = 20.09179 \text{ degree, } a = 0$$

$$= r_{o2} \sin 20.09179^\circ$$

$$\rho = 100 \sin 20.09179^\circ$$

$$= 34.3525 \text{ mm}$$

EXAMPLE 6

For gear pair of example 2, calculate relative curvature exactly at axial section


$$\text{Solution: Here } \tau = r_{o2} \sin \alpha_a$$

$$= 100 \sin 20^\circ$$

$$= 34.220 \text{ mm}$$

Conclusions

Results show that the difference in relative curvature is more prominent toward the axial plane. As shifted from the axial plane, the difference in relative curvature between screw helicoids and involute helicoids goes on reducing.

It is noticed that the amount of relative curvature at a particular section is more in involute helicoids than the screw helicoid. In terms of only Hertzian contact stress (ignoring other factors like effect of contact ratio), involute helicoids should have better strength. 

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* "A comparison of figures shown a very close agreement between the forms and positions of the projections of the contact lines of the screw helicoids and the involute helicoids on the end sections of the worms, particularly those portions of them which lie on the thread sections of worms. Therefore, except for critical drives, which should always be analysed in detail, the conditions on either type of helicoids may be used as a very close approximation to those on any other type of helicoids with a straight line generatrix"—Buckingham.

ABOUT THE AUTHOR:

PK. Banerjee is with Tata Steel Ltd., in India, working in the Growth Shop as a gearbox specialist. He can be reached at pbanerjee1@tatasteel.com or pkbanerjee6@rediffmail.com. Go online to [www.tatasteel.com/growthshop].

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